Reduced Complexity ADMM-based Schedules for LP Decoding of LDPC Convolutional Codes

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1 Introduction

Linear programming (LP) decoding of LDPC codes has been brought back to the sphere of iterative message passing processing thanks to the ADMM technique. In this work, the application of this novel decoding approach is extended to LDPC convolutional codes (LDPC-CC). A flooding based formulation of the ADMM-CC algorithm is initially provided. Then, two optimization ideas based on reordering messages updates are proposed. Simulation results show that layered schedulings are efficient to improve error correction performance while accelerating the convergence speed of the considered IEEE 1901 LDPC-CC, making it more attractive for software and hardware implementations.

2 LP Decoding of LDPC-CCs

The LP decoding problem can be formulated as:

$$\min \gamma^T v + \alpha \sum_j g(v_j)$$

subject to $T_j v = z_j, z_j \in \mathcal{P}_j, \forall j \in J$

The ADMM-based penalized decoding:

$$g(v) = -\|v - 0.5\|^2$$

3 Proposed ADMM based Schedulings for LDPC-CCs

The Flooding Scheduling (FL)

1. Initialize
2. for all $j \in J, z_j(0) = 0, x_j(0) = 0, J = 0$
3. for all $j \in J, \Pi_j(0) = \delta_v, [v_j - n_j]$
4. end for
5. end for
6. Step 1: Node Update (NU)
7. Step 2: Edge Processing
8. end for
9. Step 3: Hard Decision
10. if $\|v_j - 0.5\| > 0.5$
11. end if

The On-Demand Variable Node Activation Scheduling (OVA)

1. Initialize
2. for all $j \in J, z_j(0) = 0, x_j(0) = 0, J = 0$
3. for all $j \in J, \Pi_j(0) = \delta_v, [v_j - n_j]$
4. end for
5. end for
6. Step 1: Initial (NU)
7. Step 2: Edge Processing
8. end for
9. Step 3: Hard Decision
10. if $\|v_j - 0.5\| > 0.5$
11. end if

The Horizontal Layered Scheduling (HL)

1. Initialize
2. for all $j \in J, z_j(0) = 0, x_j(0) = 0, J = 0$
3. for all $j \in J, \Pi_j(0) = \delta_v, [v_j - n_j]$
4. end for
5. end for
6. Step 1: Initial (NU)
7. Step 2: Edge Processing
8. end for
9. Step 3: Hard Decision
10. if $\|v_j - 0.5\| > 0.5$
11. end if

4 Complexity Analysis Per Decoding Iteration

Computational Complexity

<table>
<thead>
<tr>
<th>FL</th>
<th>OVA</th>
<th>HL</th>
</tr>
</thead>
<tbody>
<tr>
<td>VN</td>
<td>CN</td>
<td>CN</td>
</tr>
<tr>
<td>CN</td>
<td>CN</td>
<td>CN</td>
</tr>
<tr>
<td>arithmetic add &amp; sub</td>
<td>$2d_a + 1$</td>
<td>$3d_a$</td>
</tr>
<tr>
<td>arithmetic multiply &amp; div</td>
<td>$1$</td>
<td>$2d_a$</td>
</tr>
<tr>
<td>abs, min, max, xor, cmp</td>
<td>$2$</td>
<td>$2d_a$</td>
</tr>
<tr>
<td>projection</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>Memory access</td>
<td>$3d_a$</td>
<td>$5d_a$</td>
</tr>
</tbody>
</table>

Memory Complexity

$$M_{FL} = M_{OVA} = 2 \times (N + m)$$

$$M_{HL} = N + 2 \times m$$

5 Experimental Results

Error Correction Performance

- HL and OVA exhibit similar BER performance.
- HL and OVA layered schedulings provide BER performance equivalent to FL with twice less iterations.
- At equivalent iteration number, HL and OVA schedulings perform slightly better than the FL.

Convergence Speed

- At the same maximum number of iterations, both HL & OVA are 1.4 times faster than FL.
- The HL & OVA succeed to halve the average number of decoding iterations.
- The spreading of reliable messages helps speeding-up the decoding convergence.

6 Conclusion & Perspectives

- The layered schedulings (HL & OVA), when applied on the iterative ADMM approach for decoding LDPC convolutional codes, improve the error correction performance while valuably speeding-up its convergence compared to the FL.
- The horizontal layered scheduling has the lowest memory complexity. For the IEEE 1901 LDPC-CC, a memory saving of about 12.5% was achieved.
- Layered schedules can be considered as promising alternatives to conventional belief propagation decoding algorithms for future software and hardware simplifications of LDPC-CC decoders.